Дата: 17.07.16 Летописец: Чернякова Лиза

$x^2 = -1$ $\varnothing$

$i^2 = -1$

$a \cdot i \quad a \in \mathbb{R}$

$z = a + bi$; $\{a, b\} \in \mathbb{R}$

$z_1 = a_1 + b_1 i$

$z_2 = a_2 + b_2 i$

$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)$

i - мнимая единица

$z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)$

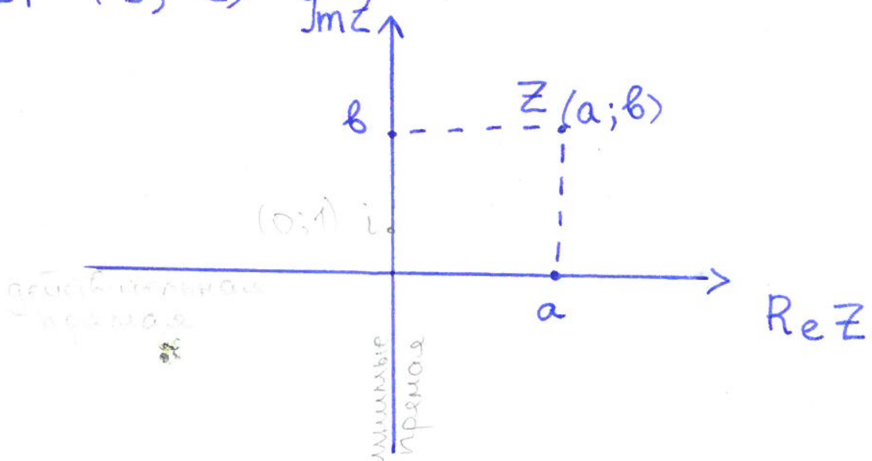
$a = \mathrm{Re}\, z$ - действительная часть числа z

$z_1 = z_2 \Leftrightarrow \begin{cases} a_1 = a_2 \\ b_1 = b_2 \end{cases}$

$b = \mathrm{Im}\, z$ - мнимая часть числа z

$z \leftrightarrow (a; b)$

$z = 3 - 2i \quad (3; -2)$



$z \in \mathbb{R} \Leftrightarrow b = 0$

$z_1 = a_1 + b_1 i$

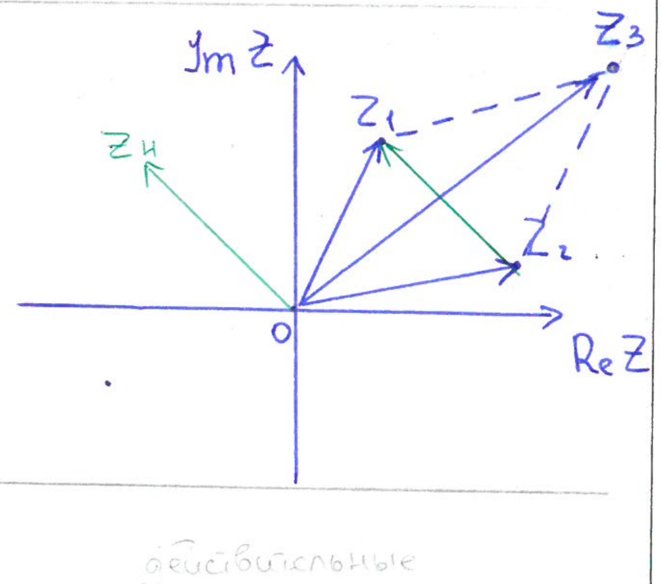
$z_2 = a_2 + b_2 i$

1) $z_3 = z_2 + z_1$

$z_3 = ?$

2) $z_4 = z_1 - z_2$

$z_4 = ?$



$z_1 = a_1 + b_1 i$

$z_2 = a_2 + b_2 i$

действительные числа

$z_1 \cdot z_2 = (a_1 + b_1 i)(a_2 + b_2 i) = \underline{a_1 a_2} + a_1 b_2 i + a_2 b_1 i + \underline{b_1 b_2 i^2} =$

$= (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) \cdot i$

$z_1 z_2 \overset{def}{=} (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i$

$a_1 \quad a_2 \qquad (a_1;0)\cdot(a_2;0) = (a_1a_2 - 0\ ;\ 0 + a_2 0) = (a_1a_2;0)$

$i = (0;1)$

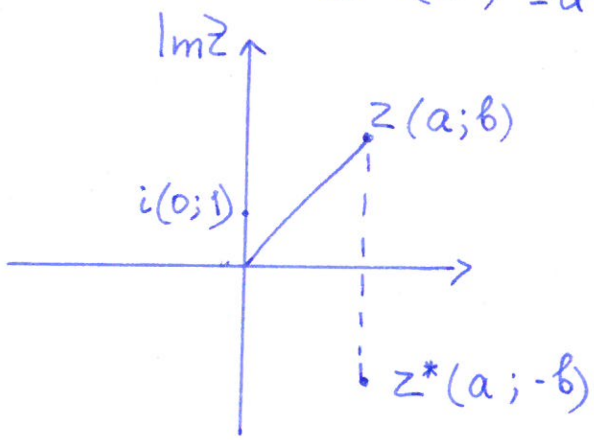
$i^2 = i\cdot i = (0;1)\cdot(0;1) = (0-1\ ;\ 0+0) = (-1;0) = -1$

$z = a + bi$

$|z| = \sqrt{a^2+b^2}$

$\bar{z} = \overset{*}{z} = a - bi$

$\bar{\bar{z}} = (z^*)^* = a + bi = z$ сопряж.



$\boxed{z = z^*} \Leftrightarrow z \in \mathbb{R}$

$|z| = |z^*|$

$z \cdot z^* = (a+bi)(a-bi) =$

$= (a;b)(a;-b) =$

$= (a^2 - b(-b); a(-b) + ab) =$

$= (a^2 + b^2; 0) = |z|^2$

$\boxed{z \cdot z^* = |z|^2}$

$kz \quad k \in \mathbb{R} \qquad k \leftrightarrow (k;0)$

$(a_1;b_1)(k;0) = (a_1k - b_1 0;\ a_1 0 + b_1k) = (a_1k;\ b_1k)$

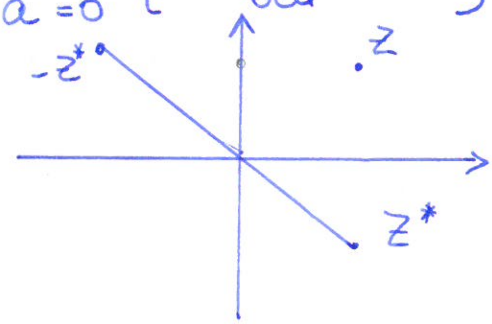
{Умножение k на z – гомотетия}

$z = -z^*$

$a + bi = -(a - bi)$

$a + bi = -a - bi$

$a = a \Leftrightarrow a = 0$ {a^* на мнимой оси}



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$Z \cdot i$

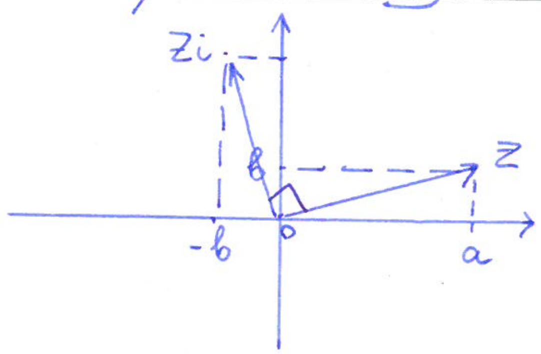
$(a+bi)i = ai + bi^2 = -b + ai$

$Z = a + bi$

$Zi = -b + ai$

$|Z| = \sqrt{a^2+b^2}$

$|Zi| = \sqrt{(-b)^2+a^2} = |Z|$

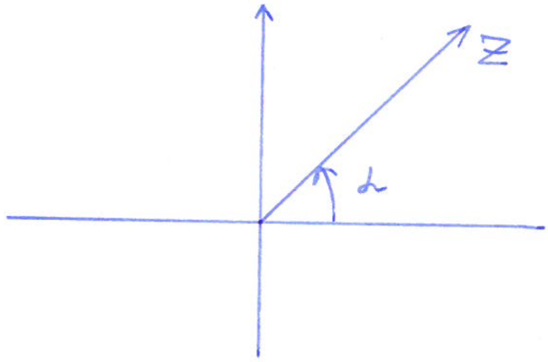


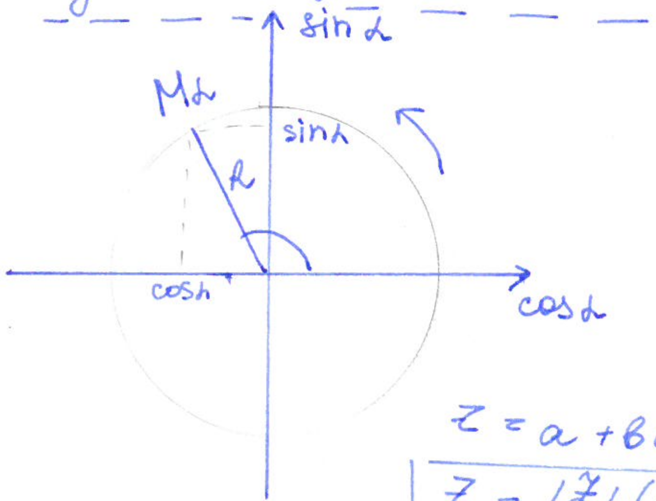
$Z \to Zi$ {при умножении на „i" → поворот на 90° относительно 0}

$|Z|$, $\arg Z = \alpha$

$\operatorname{Arg} Z$ (все значения)

$\arg Z$ $(0 \leq \arg Z < 2\pi)$





$M\alpha(R\cos\alpha; R\sin\alpha)$

$a = |Z|\cos\alpha$

$b = |Z|\sin\alpha$

$Z = a + bi = |Z|\cos\alpha + |Z|\sin\alpha\, i$

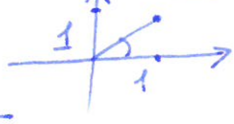
$$\boxed{Z = |Z|(\cos\alpha + i\sin\alpha)}$$

$Z \to Zi$

$$\boxed{\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}} \qquad \boxed{\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}}$$

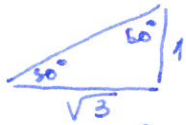
1) $Z = 1 + i$ $\quad |Z| = \sqrt{2}$

$\arg Z = \frac{\pi}{4}$



2) $Z = -\sqrt{6} + \sqrt{2}\,i$ $\quad |Z| = \sqrt{6+2}$

$\arg Z = \frac{5\pi}{6}$



$\cos\alpha = \frac{a}{\sqrt{a^2+b^2}} = \frac{-\sqrt{6}}{2\sqrt{2}} = -\frac{\sqrt{3}}{2}$ $\quad \sin\alpha = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2}$ $\Big\} = 150^\circ = \frac{5\pi}{6}$

3) $Z = 1 - \sqrt{3}\,i$ $\quad |Z| = \sqrt{1+3} = 2$

$\arg Z = \frac{5\pi}{3}$

$\left.\begin{array}{l}\cos\alpha = \frac{1}{2}\\ \sin\alpha = -\frac{\sqrt{3}}{2}\end{array}\right\} = \alpha = \frac{5\pi}{3}$

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$z_1 = a_1 + b_1 i = |z_1|(\cos\alpha_1 + i\sin\alpha_1)$

$z_2 = a_2 + b_2 i = |z_2|(\cos\alpha_2 + i\sin\alpha_2)$

$z_1 z_2 = |z_1|\cdot|z_2|\left((\cos\alpha_1\cos\alpha_2 - \sin\alpha_1\sin\alpha_2) + i(\cos\alpha_1\sin\alpha_2 + \cos\alpha_2\sin\alpha_1)\right) =$

аргумент

$= |z_1|\cdot|z_2|\left(\cos(\alpha_1+\alpha_2) + i\cdot(\sin(\alpha_1+\alpha_2)\right)$

$\operatorname{Arg} z$

$$\boxed{\begin{array}{l} |z_1 z_2| = |z_1|\cdot|z_2| \\ \operatorname{Arg}(z_1 z_2) = \operatorname{Arg} z_1 + \operatorname{Arg} z_2 \end{array}}$$

$$\arg(z_1 z_2) = \begin{cases} \arg z_1 + \arg z_2, \ldots < 2\pi \\ \arg z_1 + \arg z_2 - 2\pi, \text{ при } \arg z_1 + \arg z_2 \geq 2\pi \end{cases}$$

$z\cdot i$

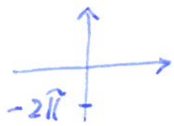
$z = |z|(\cos\alpha + i\sin\alpha) \quad i = 1(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

$z\cdot i = |z|(\cos(\alpha+\frac{\pi}{2}) + i\sin(\alpha+\frac{\pi}{2}))$

$z_1 = \sqrt{6} + \sqrt{2} i$

$z_2 = -2i$

$z_1 = 2\sqrt{2}(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$



$z_2 = 2(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})$

$z_1 z_2 = 4\sqrt{2}(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}) = 4\sqrt{2}(\frac{1}{2} - i\frac{\sqrt{3}}{2}) = 2\sqrt{2} - 2\sqrt{6}i$

Д/З 1) Поворот на 60° с помощью комплексных чисел

1 2) $z = 5 - 2i$ $H(K;2)$ задать гомотетию K - центр, 2 - коэф.

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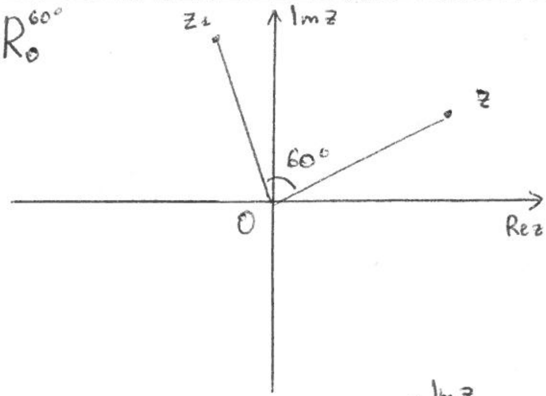
1 • $f(z)$ $R_0^{60°}$



$$f(z) = z \cdot z_2$$

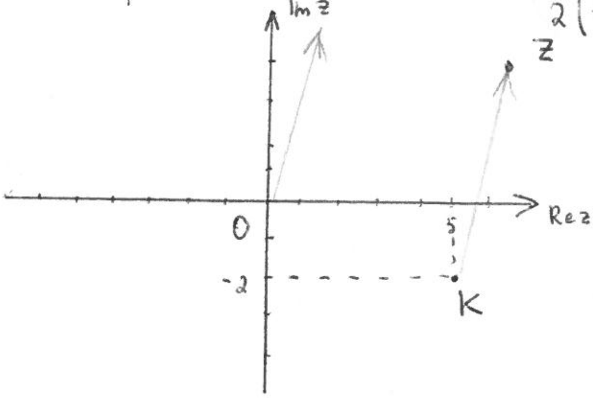
$$|z_2| = 1 \quad \arg z_2 = \frac{\pi}{3}$$

$$z_2 = |z_2|\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = \frac{1}{2} + i\,\frac{\sqrt{3}}{2}$$

$$f(z) = z\left(\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right)$$

• $H(k, 2)$

$k(5 - 2i)$



$$2(z - 5 + 2i) + 5 - 2i$$

$$H(P(z_0); k)$$

$$f(z) = k(z - z_0) + z_0 =$$

$$= kz - (k-1)z_0$$

Комплексное число задано упорядоченной парой чисел.

$z = a + bi \quad z = (a, b)$ $\left\{ a, b \in \mathbb{R} \right.$

$z = |z|(\cos\alpha + i\sin\alpha)$ $\left\{ |z| \in \mathbb{R} \right.$

$|z| = \sqrt{a^2 + b^2}$ $\quad \alpha \in [0; 2\pi)$

$z_1 = a_1 + b_1 i \qquad z_1 \pm z_2 = z_3 \quad z_3 = a_1 \pm a_2 + (b_1 \pm b_2)i$

$z_2 = a_2 + b_2 i \qquad z_1 \cdot z_2 = z_4 \quad z_4 = a_1a_2 - b_1b_2 + (a_1b_2 + a_2b_1)$

$z_1 = |z_1|(\cos\alpha_1 + i\sin\alpha_1)$

$z_2 = |z_2|(\cos\alpha_2 + i\sin\alpha_2)$

$z_4 = |z_1||z_2|(\cos(\alpha_1 + \alpha_2) + i\sin(\alpha_1 + \alpha_2))$

$z_4 = (|z_1||z_2|;\ \underbrace{Arg(z_1 \cdot z_2)}_{Arg\, z_1 + Arg\, z_2})$

$z^* = a - bi$

$$\frac{z_1}{z_2} = \frac{a_1 + b_1 i}{a_2 + b_2 i} = \frac{(a_1 + b_1 i)(a_2 - b_2 i)}{|z_2|^2} = \frac{a_1a_2 + b_1b_2 - (a_1b_2 - a_2b_1)i}{|z_2|^2}$$

$$\frac{1}{\sqrt{5} - \sqrt{3}} + \frac{1}{\sqrt{7} - \sqrt{5}} + \frac{1}{\sqrt{9} - \sqrt{7}} = \frac{\sqrt{5} + \sqrt{3}}{5 - 3} + \frac{\sqrt{7} + \sqrt{5}}{7 - 5} + \frac{\sqrt{9} + \sqrt{7}}{9 - 7} = \frac{\sqrt{3} + 2\sqrt{5} + 2\sqrt{7} + \sqrt{9}}{2} =$$

$$= \sqrt{5} + \sqrt{7} + \frac{\sqrt{3} + 3}{2}$$

$$\frac{z_1}{z_2} = \frac{|z_1|(\cos\alpha_1 + i\sin\alpha_1)}{|z_2|(\cos\alpha_2 + i\sin\alpha_2)} = \frac{|z_1|}{|z_2|} \cdot \frac{\cos\alpha_1\cos\alpha_2 + \sin\alpha_1\sin\alpha_2 - (\cos\alpha_1\sin\alpha_2 - \cos\alpha_2\sin\alpha_1)i}{\cos^2\alpha_2 + \sin^2\alpha_2} =$$

$$= \frac{|z_1|}{|z_2|} \cdot \frac{\cos(\alpha_1 - \alpha_2) + i\cdot\sin(\alpha_1 - \alpha_2)}{1}$$

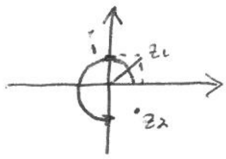
$$\frac{z_1}{z_2} = z_5;\ |z_5| = \frac{|z_1|}{|z_2|} \quad Arg\, z_5 = Arg\, z_1 - Arg\, z_2$$

Дата: 18.06.16 Летописец: Баранова Людмила

$$\arg\frac{z_1}{z_2} = \begin{cases} \arg z_1 - \arg z_2, & \arg z_1 \ge \arg z_2 \\ \arg z_1 - \arg z_2 + 2\pi, & \arg z_1 < \arg z_2 \end{cases} \qquad \frac{z_1}{z_2} = \frac{z_1 z_2^*}{|z_2|^2}$$

• $z_1 = 1+i$ $z_2 = 1-i$ $\frac{z_1}{z_2}$ – ?

$$\frac{z_1}{z_2} = \frac{1+i}{1-i} = \frac{1\cdot 1 + 1\cdot(-1) - (1\cdot(-1) - 1\cdot 1)i}{1^2+1^2} = \frac{\cancel{2}i}{\cancel{2}} = i$$



• $z_1 = 1+\sqrt{3}i$ $z_2 = -\sqrt{3}-i$ $z_6 = \frac{z_1}{z_2}$ z_6 – ?

$$z_6 = \frac{z_1}{z_2} = \frac{(1+\sqrt{3}i)(-\sqrt{3}+i)}{(-\sqrt{3}-i)(-\sqrt{3}+i)} = \frac{-\sqrt{3}-\sqrt{3}+(1-3)i}{4} = \frac{-\sqrt{3}-i}{2}$$

$z_1 = |z_1|(\cos\alpha_1 + i\sin\alpha_1)$

$\frac{|z_1|}{|z_2|} = \frac{\sqrt{2}}{\sqrt{2}} = 1$ $|z_5| = 1$

$\operatorname{Arg} z_1 - \operatorname{Arg} z_2 = -\frac{3}{2}\pi$

$z_5 = 1\left(\cos\left(-\frac{3\pi}{2}\right) + \sin\left(-\frac{3\pi}{2}\right)i\right) = i$

$\forall z = a+bi$

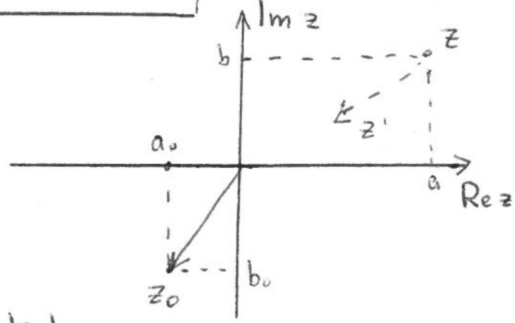
1. $z + z_0$

 $z_0 = a_0 + b_0 i$

2. $z\cdot k, k\in\mathbb{R}$
3. $z\cdot i$
4. $z\cdot z_0 = z_1$

$|z_1| = |z|\cdot|z_0|$

$\operatorname{Arg} z_1 = \operatorname{Arg} z + \arg z_0$



$z^n = (a+bi)^n$

$z = |z|(\cos\alpha + \sin\alpha\cdot i)$

$z^2 = z\cdot z = |z|^2(\cos 2\alpha + i\sin 2\alpha)$

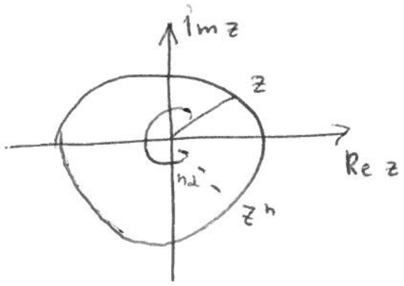
$z^3 = z^2\cdot z = |z|^3(\cos 3\alpha + i\sin 3\alpha)$

. . .

$$\boxed{z^n = |z|^n(\cos n\alpha + i\sin n\alpha)}$$

$n\in\mathbb{N}$ формула Муавра

$|z| = 1$ $z^n = \cos n\alpha + i\sin n\alpha$



• i^{2016} – ?

$|i| = 1$ $\cos\alpha = 0$ $\sin\alpha = 1$

$i^2 = 1$

$i^3 = -1$

$i^4 = 1$

...

$i^{2016} = 1$

• $z = -1+\sqrt{3}i$ z^{20} – ?

$|z| = \sqrt{1+3} = \sqrt{4} = 2$ $\cos\alpha = -\frac{1}{2}$ $\sin\alpha = \frac{\sqrt{3}}{2}$ $\alpha = \frac{2\pi}{3}$ $20\alpha = \frac{40\pi}{3}$

$\cos\frac{40\pi}{3} = \cos\frac{4\pi}{3}$ $\sin\frac{40\pi}{3} = \cancel{\frac{\sqrt{3}}{2}}\ \sin\frac{4\pi}{3}$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha = \cancel{2}\left(-\frac{1}{2}\right)\cdot\frac{\sqrt{3}}{\cancel{2}} = -\frac{\sqrt{3}}{2}$

$z^{20} = 2^{20}\left(\left(-\frac{1}{2}\right) + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

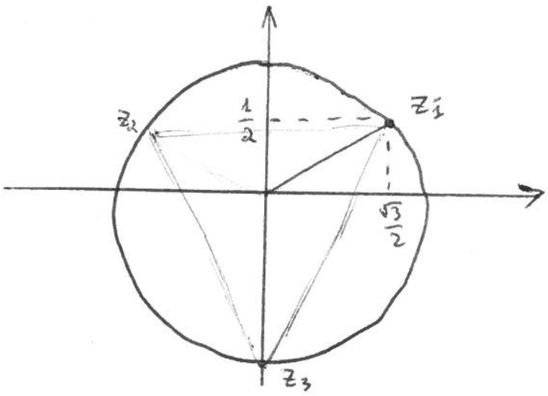
• $z^3 = i \quad z^3 = |z|^3 (\cos 3\alpha + i \sin 3\alpha) = 1 (\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) = 1 (\cos \frac{5\pi}{2} + i \sin \frac{5\pi}{2}) =$

$|z|^3 = 1 \quad 3\alpha = \frac{\pi}{2} \quad |z| = 1 \quad \alpha = \frac{\pi}{6}$

$= 1 (\cos \frac{9\pi}{2} + i \sin \frac{9\pi}{2}) =$

$\alpha_1 = \frac{\pi}{6} \quad \alpha_2 = \frac{5\pi}{6} \quad \alpha_3 = \frac{3\pi}{2}$

$= 1 (\cos \frac{13\pi}{2} + i \sin \frac{13\pi}{2})$



$z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2} i \quad z_2 = -\frac{\sqrt{3}}{2} + \frac{1}{2} i$

$z_3 = -i$

$z^4 = -16 \qquad |z|^4 (\cos 4\alpha + i \sin 4\alpha) = 16 (\cos \pi + i \sin \pi)$

$|z|^4 = 16 \quad |z| = 2 \quad 4\alpha = \left[\begin{array}{l} \pi \\ 3\pi \\ 5\pi \\ 7\pi \\ \vdots \end{array}\right. \quad \alpha = \left[\begin{array}{l} \frac{\pi}{4} \\ \frac{3\pi}{4} \\ \vdots \end{array}\right.$

$z = |z| (\cos \alpha + i \sin \alpha)$

$\sqrt[n]{z} = \sqrt[n]{|z|} \left(\cos \frac{\alpha + 2\pi k}{n} + i \sin \frac{\alpha + 2\pi k}{n}\right) \qquad k = \overline{0, 1, \ldots, n-1}$

$\sqrt[4]{-16} = \pm\sqrt{2} \pm i\sqrt{2}$

→ n разных чисел

Д/З 1) $(1-i)^{30}$

2 2) $\frac{\sqrt{-1}}{\sqrt{1}} = \frac{\sqrt{1}}{\sqrt{-1}} \qquad \sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}}$

$(\sqrt{1})^2 = (\sqrt{-1})^2$

$1 = -1$

Дата: 19.06.16

Летописец: Чернякова Лиза

Повторение:

1) $z_1 \pm z_2$ <=> параллелограм

2) $z_1 \cdot z_1$ <=> поворотная гомотетия

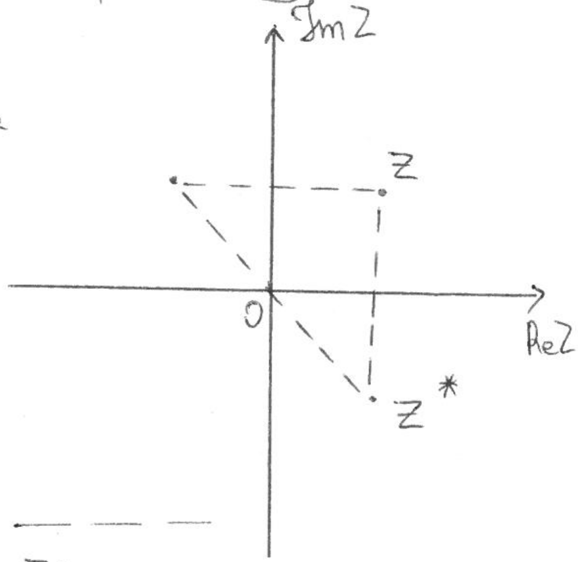
3) $\frac{z_1}{z_2} = \frac{z_1 z_2^*}{|z_2|^2}$

4) z^*

5) $|z|^2 = z \cdot z^*$

6) $z = z^* \Leftrightarrow z \in \mathbb{R}$

7) $z = -z^*$ $z \in i\mathbb{R}$?

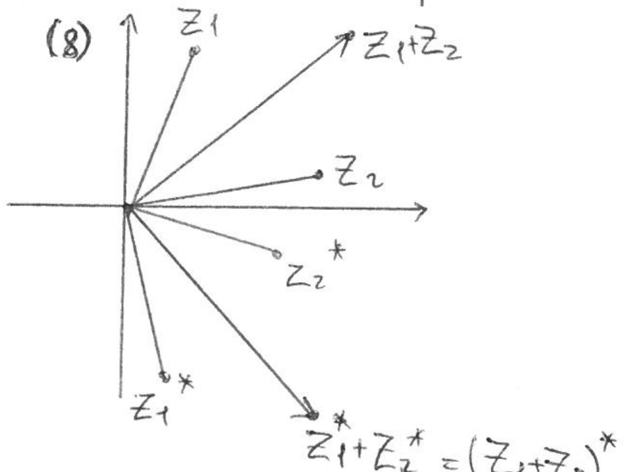


8) $(z_1 + z_2)^* = z_1^* \pm z_2^*$

9) $(z_1 z_2)^* = z_1^* \cdot z_2^*$ =>

10) $\left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*}$

=> $(tz)^* = tz^*$, $t \in \mathbb{R}$



(9) $z_1 = |z_1| \cdot (\cos\alpha_1 \cdot i\sin\alpha_1)$

$z_1^* = |z_1| \cdot (\cos\alpha_1 + i\cdot\sin\alpha_1)$

$z_2^* = |z_2| \cdot (\cos\alpha_2 + i\sin(-\alpha_2))$

$z_1^* z_2^* = |z_1||z_2| \cdot \cos(\alpha_1 + \alpha_2) + \sin(-\alpha_1 + \alpha_2) =$

$= |z_1||z_2| (\cos(\alpha_1 + \alpha_2) - i\sin(\alpha_1 + \alpha_2))$

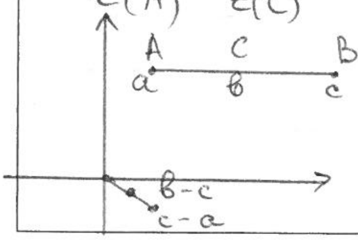
$(z_1 z_2)^* = |z_1||z_2| \cdot (\cos(\alpha_1 + \alpha_2) - i\sin(\alpha_1 + \alpha_2))$

(10) $\left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*}$, $\left(\frac{z_1}{z_2}\right)^* = \left(\frac{z_1 z_2^*}{|z_2|^2}\right)^* = \left(\frac{z_1}{|z_2|^2}\right)^* \cdot (z_2^*)^* = \left(\frac{1}{|z_2|^2} \cdot z_1\right)^* \cdot z_2 =$
$= \left(\frac{1}{|z_2|^2} z_1\right)^* \cdot z_2$

I A C B $\frac{AC}{CB} = \lambda > 0$ Найти т. C

$z(A)$ $z(C)$ $z(B)$ $a, b, c \in \mathbb{C}$

AB: луч, прямая, отр
AC – длина
$\overrightarrow{AC}$ – вектор



$c - a$ $b - c$

$c - a = \lambda(b - c)$ $c - a = \lambda b - \lambda c$

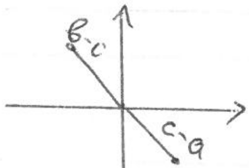
$c(1 + \lambda) = a + \lambda b$ $\boxed{c = \frac{a + \lambda b}{1 + \lambda}}$

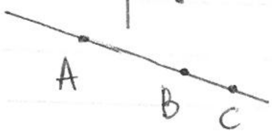
Дата: 19.06.16 Летописец: Чернякова Лиза

$\frac{AC}{BC} = k > 0$ $\quad \overline{BC} = \overline{CB}$ $\quad (c-a) = (b-c)(-k)$

$\lambda = -k$

$c = \frac{a+\lambda b}{1+\lambda}$ $\quad \lambda < 0$





$-1 < \lambda < 0$ $\quad \to$ т C слева от т A

$\lambda = -1$ $\quad \varnothing$ (не подходит в уравн.)

$\lambda = -1 \Rightarrow k = 1$ $\varnothing$

$\lambda = 0$ $\quad c = A$

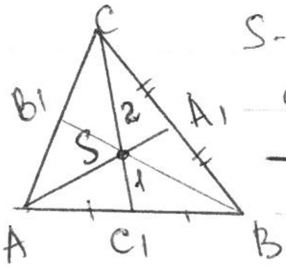
$c = \frac{a+\lambda b}{1+\lambda}$ $\quad \lambda \neq -1$ $\quad \lambda \in \mathbb{R}$

Уравнение прямой AB

Середина отрезка

$c = \frac{a+b}{2}$ $\quad \lambda = 1$

Центр тяжести Δ



S - середина

$c_1 = \frac{a+b}{2}$ $\quad S = \frac{c + 2\left(\frac{a+b}{2}\right)}{3} = \frac{a+b+c}{3}$

Д/З 1. Найти центр тяжести четугольника!

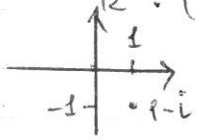
3 $\quad \frac{a+b+c+d}{4} = S$

2. Найти центр тяжести треугольника без уравнения медиан.

---

2 1) $(1-i)^{30} = \left(\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i \cdot \sin\left(-\frac{\pi}{4}\right)\right)\right)^{30} = 2^{15}\left(\cos\left(-\frac{15\pi}{2}\right) + i \cdot \sin\left(-\frac{15\pi}{2}\right)\right) =$

$= 2^{15} \cdot \left(\cos\frac{\pi}{2} + i \cdot \sin\frac{\pi}{2}\right) = 2^{15} \cdot i$



2) $\sqrt[3]{i} = \left\{ \pm\frac{\sqrt{3}}{2} + \frac{1}{2}i \; ; -i \right\}$

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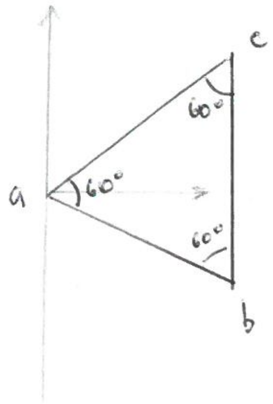


Geg: a; b

Ges: c

$\cos\alpha + i\sin\alpha = E(\alpha)$

$\cancel{c = (a-b)}$

$c = a + (b-a)\left(E\left(\frac{\pi}{3}\right)\right)$

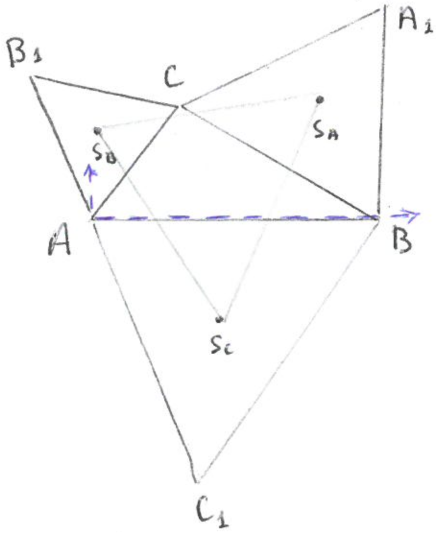
$$|E(\alpha)|^2 = [\operatorname{Re} E(\alpha)]^2 + [\operatorname{Im} E(\alpha)]^2 = 1$$

$\operatorname{Re} E(\alpha) = \cos\alpha$ $\operatorname{Im} E(\alpha) = \sin\alpha$

$z \to z E(\alpha)$ Поворот вокруг начала координат на угол α.

$E\left(\frac{\pi}{2}\right) = 0 + i = i$ $E\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + \frac{i}{2}$ $E\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{i\sqrt{3}}{2}$ $E\left(-\frac{\pi}{2}\right) = 0 + (-i) = -i$

Треугольники Наполеона



ΔA_1BC, ΔAB_1C, ΔABC_1 — равносторонние.

S_A, S_B, S_C — ц. тяжести этих Δ

Доказать: $\Delta S_AS_BS_C$ — равносторонний.

$\operatorname{Im} b = 0$ $b \in \mathbb{R}$ b, c sind gegeben

$c_1 = bE\left(-\frac{\pi}{3}\right)$ $b_1 = cE\left(\frac{\pi}{3}\right)$ $a_1 = (b-c)E\frac{\pi}{3} + c$

$\Delta S_AS_BS_C$ равностор. $\Longleftrightarrow S_A = (S_B - S_C)E\left(\frac{\pi}{3}\right) + S_C$

$S_A = \frac{a_1+b+c}{3} = \frac{(b-c)E\left(\frac{\pi}{3}\right)+2c+b}{3}$

$S_B = \frac{cE\left(\frac{\pi}{3}\right)+c}{3}$

$S_C = \frac{b+bE\left(-\frac{\pi}{3}\right)}{3}$

$$(S_B - S_C)E\left(-\tfrac{\pi}{3}\right) + S_C - S_A = 0 \quad ?$$

$$\frac{cE\left(\frac{\pi}{3}\right)+c-b-bE\left(-\frac{\pi}{3}\right)}{3}E\left(-\tfrac{\pi}{3}\right) + \frac{b+bE\left(-\frac{\pi}{3}\right)-2c-b-(b-c)E\left(\frac{\pi}{3}\right)}{3} = 0 \quad ?$$

$$cE\left(\tfrac{\pi}{3}\right)E\left(-\tfrac{\pi}{3}\right) + (c-b)E\left(-\tfrac{\pi}{3}\right) - bE^2\left(-\tfrac{\pi}{3}\right) + b + bE\left(-\tfrac{\pi}{3}\right) - 2c - b - (b-c)E\tfrac{\pi}{3} =$$

$$= c - 2c + (c-b)\left[E\left(-\tfrac{\pi}{3}\right) + E\left(\tfrac{\pi}{3}\right)\right] - bE\left(-\tfrac{2\pi}{3}\right) + bE\left(-\tfrac{\pi}{3}\right) =$$

$$= -c + (c-b)\left[E^*\left(\tfrac{\pi}{3}\right) + E\left(\tfrac{\pi}{3}\right)\right] + b\left[E\left(-\tfrac{\pi}{3}\right) - E\left(-\tfrac{2\pi}{3}\right)\right] = \qquad -E\left(-\tfrac{2\pi}{3}\right) = E\left(\tfrac{\pi}{3}\right)$$

$$= -c + 2(c-b)\operatorname{Re}E\left(\tfrac{\pi}{3}\right) + b\cdot 2\operatorname{Re}E\left(-\tfrac{\pi}{3}\right) =$$

$$= -c + \cancel{2}(c-b)\cdot\frac{1}{\cancel{2}} + b\cdot\cancel{2}\cdot\frac{1}{\cancel{2}} = \cancel{-c} + \cancel{c} - \cancel{b} + b = 0 \quad \blacksquare$$

△$S_AS_BS_C$ – внешний Наполеонов треугольник

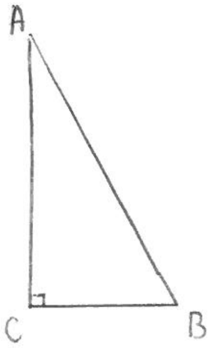
△A_2BC, △AB_2C, △ABC_2 – равносторонние треугольники (внутрь)

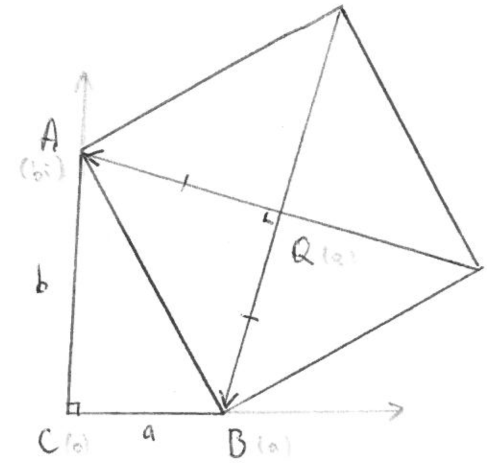
T_A, T_B, T_C – ц. тяжести этих △.

△$T_AT_BT_C$ – внутренний Наполеонов треугольник

Центры тяжести △ABC, △$S_AS_BS_C$, △$T_AT_BT_C$

△$S_AS_BS_C$ → △$T_AT_BT_C$ / △$T_AT_BT_C$ → △$S_AS_BS_C$ – ???





$\overrightarrow{QA} = bi - q$

$\overrightarrow{QB} = a - q$

$\overrightarrow{QA} \leftrightarrow bi - q$

$\overrightarrow{QB} \leftrightarrow a - q$

$\overrightarrow{QB} = i\,\overrightarrow{QA}$

$a - q = i(bi - q)$ $\quad i^2 = -1$

$a - q = -b - iq$

$a + b = q(1 - i)$

$q = \frac{a+b}{1-i}$ $\quad |q|^2 = qq^*$

$|q|^2 = \frac{a+b}{1-i} \cdot \frac{a+b}{1+i} = \frac{(a+b)^2}{1+1} = \frac{(a+b)^2}{2}$

$|q| = \frac{a+b}{\sqrt{2}} = CQ$

$\omega(O(z_0), R)$ $\quad M(z)$ $\quad M \in \omega \Longleftrightarrow MO = R$ $\quad |t|^2 = tt^*$

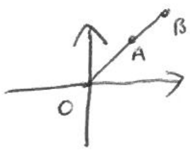
$|z - z_0|^2 = R^2$

$(z - z_0)(z - z_0)^* = R^2$

$\boxed{(z - z_0)(z^* - z_0^*) = R^2}$

Параллельность

$O(0)$ $A(a)$ $B(b)$ $\quad a \in \mathbb{C}$ $b \in \mathbb{C}$



$\left[\begin{array}{l} \arg a - \arg b = 0 \\ \arg a - \arg b = \pm\pi \end{array}\right.$

$t \in \mathbb{R} \Longleftrightarrow t = t^*$

$\arg \frac{a}{b} \in \mathbb{R} \Longleftrightarrow \frac{a^*}{b^*} = \frac{a}{b}$

$a^*b = ab^*$ колинеарны

$A(a)$ $B(b)$ $C(c)$ $D(d)$ $\quad \{a; b; c; d\} \in \mathbb{C}$

AB и CD ? $\quad \underset{b-a}{\overrightarrow{AB}}$ $\underset{d-c}{\overrightarrow{CD}}$ $\quad (b-a)^*(c-d) = (b-a)(c-d)^*$

$(b^* - a^*)(c-d) = (b-a)(c^* - d^*)$

$\{A, B, C, D\} \subset \omega(O(0); R=1)$ $\quad zz^* = 1 \Rightarrow z^* = \frac{1}{z}$

$\left(\frac{1}{b} - \frac{1}{a}\right)(d-c) = (b-a)\left(\frac{1}{d} - \frac{1}{c}\right)$

$\frac{(a-b)(d-c)}{ab} = \frac{(b-a)(c-d)}{cd}$ $\quad \boxed{ab = cd}$

Перпендикулярность

$OA \perp OB \quad O(0)\ A(a)\ B(b)$

$\arg a - \arg b = \pm \frac{\pi}{2}$

$\left[\begin{array}{l} \arg \frac{a}{b} = \frac{\pi}{2} \\ \arg \frac{a}{b} = \frac{3\pi}{2} \end{array}\right.$ $\quad \operatorname{Re} \frac{a}{b} = 0$ $\quad \frac{a}{b} = -\left(\frac{a}{b}\right)^*$ $\quad \frac{a}{b} = -\frac{a^*}{b^*}$

$\boxed{a^*b + ab^* = 0}$

$A(a)\ B(b)\ C(c)\ D(d)\quad AB \perp CD\ ?!$

$\vec{AB}\ \ b-a \quad \vec{CD}\ \ d-c$

$(b-a)^*(d-c) + (b-a)(d-c)^* = 0$

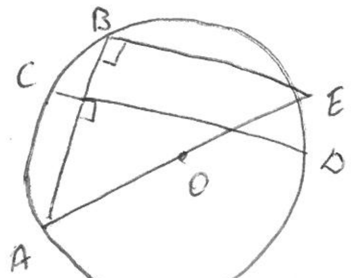
$(b^*-a^*)(d-c) + (b-a)(d^*-c^*) = 0$

$\{A, B, C, D\} \subset \omega(O(0); R=1)$ $\quad a^* = \frac{1}{a}\ \ b^* = \frac{1}{b}\ \ c^* = \frac{1}{c}\ \ d^* = \frac{1}{d}$

$\left(\frac{1}{b} - \frac{1}{a}\right)(d-c) + (b-a)\left(\frac{1}{d} - \frac{1}{c}\right) = 0$

$\frac{(a-b)(d-c)}{ab} + \frac{(b-a)(c-d)}{cd} = 0$ $\quad \frac{1}{cd} + \frac{1}{ab} = 0$ $\quad \boxed{ab = -cd}$

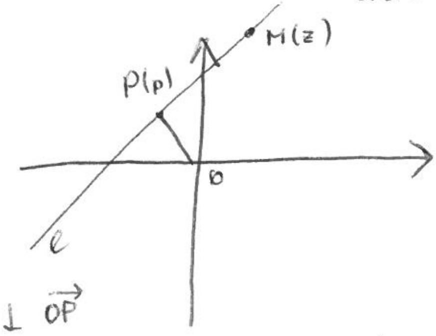
$ab + cd = 0$



BE и CD

$cd = be$

$e = -a$



$p \neq 0$

$M \in l \Leftrightarrow \vec{PM} \perp \vec{OP}$

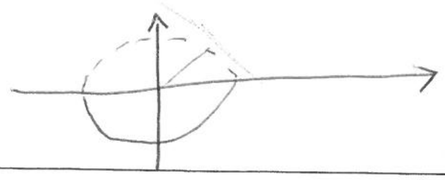
$\vec{PM}\ \ z-p \quad \vec{OP}\ \ p$

$M \in l \Leftrightarrow (z-p)p^* + (z^*-p^*)p = 0$

$zp^* - pp^* + zp^* - pp^* = 0$ $\quad \boxed{zp^* + z^*p = 2pp^*}$ $\quad p \neq 0$

$P(p) \in \omega(O(0); R=1)$ $\quad pp^* = 1$

$\boxed{zp^* + z^*p = 2}$



$AB \parallel CD \Longleftrightarrow \dfrac{a-b}{a^*-b^*} = \dfrac{c-d}{c^*-d^*}$

$A, B, C \in l \Longleftrightarrow AB \parallel AC \Longleftrightarrow \dfrac{a-b}{a^*-b^*} = \dfrac{a-c}{a^*-c^*}$

$a \cdot a^* - a \cdot c^* - b \cdot a^* + b \cdot c^* - a^* a + a^* c + b^* a - b^* c = 0$

$a(b^* - c^*) + b(c^* - a^*) + c(a^* - b^*) = 0$

$A, B, z \in l \Longleftrightarrow a(b^* - z^*) + b(z^* - a^*) + z(a^* - b^*) = 0$

$\omega : z \cdot z^* = 1$

$A \in \omega \Longleftrightarrow a^* = \dfrac{1}{a} \qquad B \in \omega \Longleftrightarrow b^* = \dfrac{1}{b}$

$a\left(\dfrac{1}{b} - z^*\right) + b\left(z^* - \dfrac{1}{a}\right) + z\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = 0$

$\dfrac{a}{b} - a \cdot z^* + b \cdot z^* - \dfrac{b}{a} + \dfrac{z}{a} - \dfrac{z}{b} = 0$

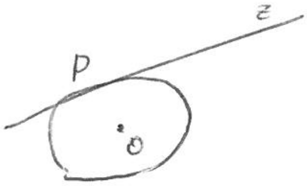
$\dfrac{a^2 - a^2 b z^* + a b^2 \cdot z^* - b^2 + z \cdot b - za}{a \cdot b} = 0$

$(a-b)(a+b) - abz^*(a-b) - z(a-b) = 0$

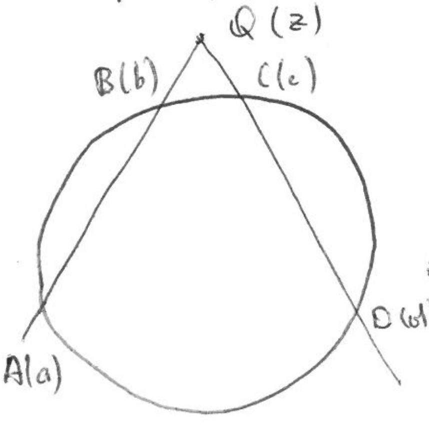
$(a-b)(a+b-abz^*-z) = 0$

$\boxed{a+b-abz^*-z=0}$

$OA \perp OB \Longleftrightarrow ab^* + a^*b = 0$



$PZ: pz^* + p^*z = 2$



$AB: a+b-abz^*-z=0$

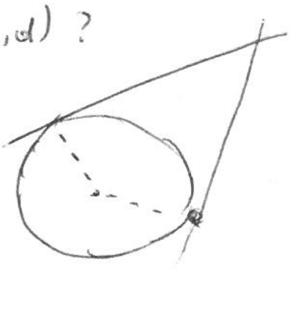
$CD: c+d-cdz^*-z=0$

$a+b-(c+d)-(ab-cd)z^*=0$

$z_0^* = z^* = \dfrac{a+b-(c+d)}{ab-cd} \qquad \omega: z \cdot z^* = 1$

$z_0 = \dfrac{a^*+b^*-(c^*+d^*)}{a^*b^*-c^*d^*} \qquad a^* = \dfrac{1}{a} \qquad a \cdot a^* = 1$

$\cancel{z_0 = \dfrac{ab-cd}{a+b-(c+d)}} \qquad z_0 = f(a,b,c,d)\ ?$



$\omega: z \cdot z^* = 1 \qquad z_1 = \dfrac{2ab}{a+b}$

$H(a,b) = \dfrac{2}{\frac{1}{a} + \frac{1}{b}}$