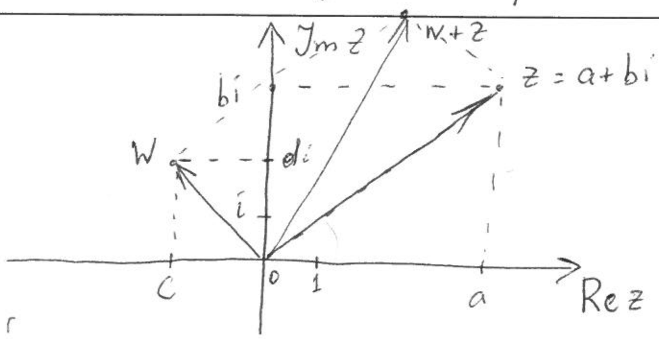
Дата: 23. 09. 16 Летописец: Людмила Баранова

$i^2 = -1$

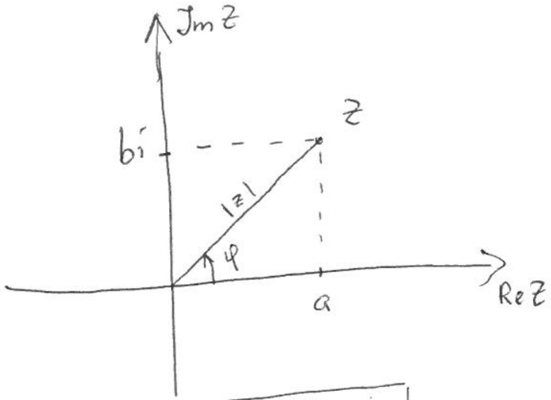
$a + bi = z$

$a = \operatorname{Re} z$

$b = \operatorname{Im} z$



$w + z = (a + c) + (b + d)i$



$0 \le \varphi \le \arg z < 2\pi$

$|z|$ — модуль z

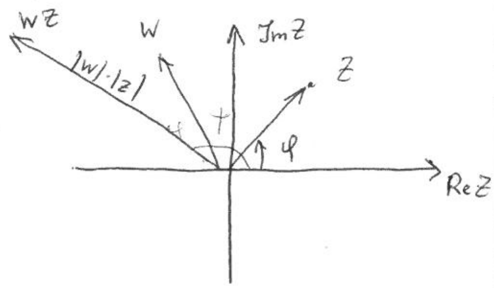
$a = |z| \cdot \cos\varphi$

$b = |z| \cdot \sin\varphi$

$|z| = \sqrt{a^2 + b^2}$

$\varphi = \operatorname{arctg}\left(\frac{b}{a}\right)$

$\boxed{\cos\varphi + i\sin\varphi = e^{i\varphi}}$



$z = a + bi = |z|(\cos\varphi + i\sin\varphi) = |z| \cdot e^{i\varphi}$

$w \cdot z = |w| \cdot |z| \cdot e^{i(\varphi + \psi)}$

$|wz| = |w| \cdot |z|$

$\arg(wz) = \arg(w) + \arg(z)$

$\frac{z}{w} = \frac{|z|}{|w|} e^{i(\varphi - \psi)}$

$z^n = |z|^n \cdot e^{in\varphi}$ $\quad |z^n| = |z|^n \quad \arg(z^n) = (\arg z)^n$

$\sqrt[n]{z} = w \quad w^n = z \Leftrightarrow |w| = \sqrt[n]{|z|}$

$n \cdot \arg w = (\arg z + 2\pi k) \,|\, : n$

$\arg w_k = \frac{1}{n} \arg z + 2\pi \frac{k}{n}$

$\sqrt[3]{1_c}$ — комплексное число $\quad 1_c = 1 e^{i \cdot 0}$

$|1_c| = 1 \quad |w| = \sqrt[3]{1} = 1$

$\arg 1 = 0 \quad \arg w_k = \frac{1}{3} \arg 1 + \frac{2\pi k}{3} \qquad k = 0, 1, 2$

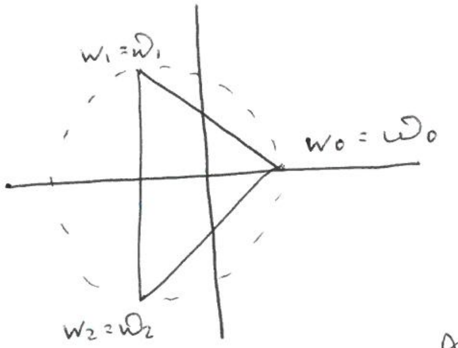
$\arg w_0 = 0$

$\arg w_1 = \frac{2\pi}{3}$

$\arg w_2 = \frac{4\pi}{3}$

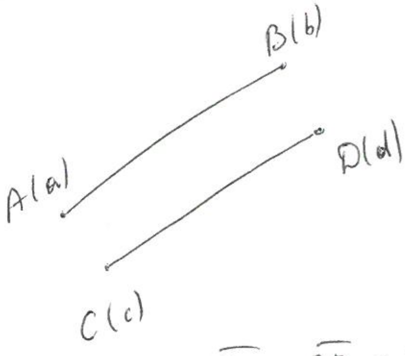
$w_0 = 1 \quad w_1 = 1 \cdot e^{i\frac{2}{3}\pi} = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

$w_2 = 1 \cdot e^{i\frac{4}{3}\pi} = \cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$



Равносторонний треугольник

$w_0 = \omega_0 \quad w_1 = \omega_1 \quad w_2 = \omega_2$



$\vec{AB}\ (b-a)$

$\vec{CD}\ (d-c)$

$\vec{AB} \parallel \vec{CD} \iff$

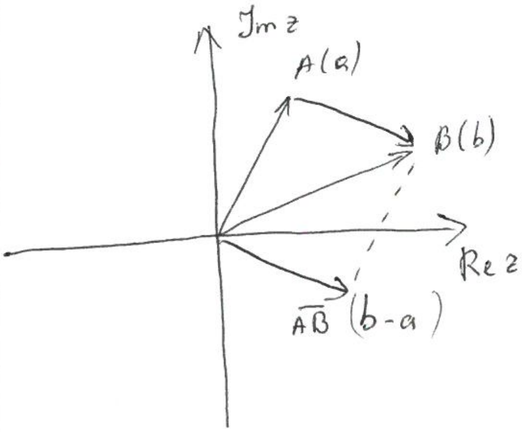
$\arg(b-a) = \arg(d-c)$

oder

$\vec{AB} = t\,\vec{CD}$

$\iff \boxed{\frac{b-a}{d-c} = \left(\frac{b-a}{d-c}\right)^*}$

$\frac{b-a}{d-c} \in \mathbb{R}$



$\vec{OA} + \vec{AB} = \vec{OB}$

$a \quad z \quad b$

$z = b - a$

1 $\quad A, B, D \in l \leftarrow$ прямая

$\vec{AB}, \vec{AD}$

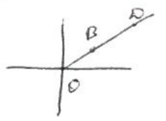
$\frac{b-a}{d-a} = \left(\frac{b-a}{d-a}\right)^* \iff (b-a)(d-a)^* = (b-a)^*(d-a)$

$\boxed{\left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*} \qquad (z_1 z_2)^* = z_1^* \cdot z_2^*}$

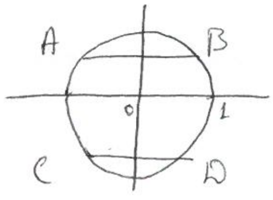
$b(d^* - a^*) - \underline{a}(d^* - a^*) - d(b^* - a^*) + \underline{a}(b^* - a^*) = 0$

$a(b^* - d^*) + b(d^* - a^*) + d(a^* - b^*) = 0$

$O, B, D \in l$



$\frac{b}{d} = \left(\frac{b}{d}\right)^* \iff bd^* = b^*d$



$A, B, C, D \in K(0; 1)$ круг

$|a| = 1 \Rightarrow a = e^{i\alpha}$

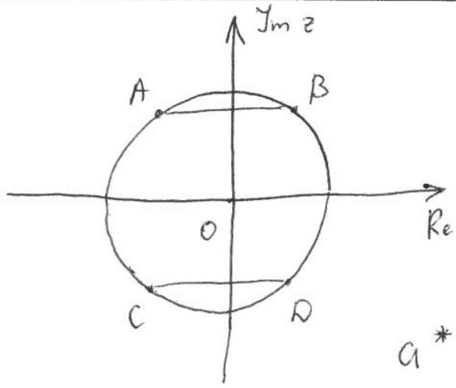
$a^* = \cos\alpha - i\sin\alpha = \cos(-\alpha) + i\sin(-\alpha)$

$a^* = e^{-i\alpha} = \frac{1}{a}$

Дата: 23.09.16 Летописец: Людмила Баранова

23.09.16 после занятий математикой в лицее мы посетили Национальную филармонию в 19:00. Там проходил концерт для виолончели по случаю 110-летия от дня рождения российского композитора и пианиста Дмитрия Шостаковича. Мы провели там около двух часов, слушая музыку. Концерт состоял из двух частей. После него мы прошлись пешком по вечернему Киеву и вернулись домой на метро. Вечер в филармонии прошёл хорошо.

Дата: 24.09.16 Летописец: Людмила Баранова



$$A, B, C, D \in K(O, r=1)$$

$$\overline{AB} \parallel \overline{CD} \iff \frac{b-a}{d-c} = \left(\frac{b-a}{d-c}\right)^* \iff$$

$$\iff \frac{b-a}{b^*-a^*} = \frac{d-c}{d^*-c^*}$$

$$a^* = \frac{1}{a} \quad b^* = \frac{1}{b} \quad c^* = \frac{1}{c} \quad d^* = \frac{1}{d}$$

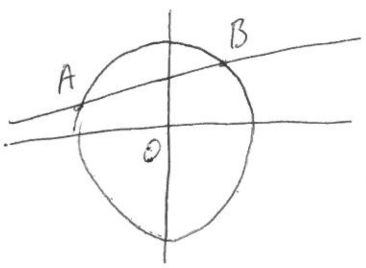
$$\frac{b-a}{\frac{1}{b}-\frac{1}{a}} = \frac{d-c}{\frac{1}{d}-\frac{1}{c}} \iff \frac{b-a}{\frac{a-b}{ab}} = \frac{d-c}{\frac{c-d}{cd}} \iff \boxed{ab = cd}$$

$$A, B, D \in l : a(b^* - d^*) + b(d^* - a^*) + d(a^* - b^*) = 0$$

$$D \to Z : a(b^* - z^*) + b(z^* - a^*) + z(a^* - b^*) = 0$$

(Уравнение прямой, проходящей через две заданные точки)

$$\iff \boxed{z(a^* - b^*) + z^*(b-a) + ab^* - a^*b = 0}$$



$$a^* = \frac{1}{a} \quad b^* = \frac{1}{b}$$

$$z\left(\frac{1}{a} - \frac{1}{b}\right) + z^*(b-a) + a\cdot\frac{1}{b} - \frac{1}{a}b = 0$$

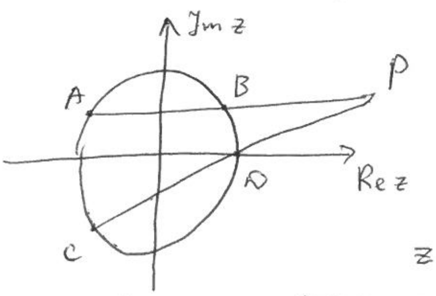
(уравнение секущей)

$$z \cdot \frac{b-a}{ab} + z^*(b-a) + \frac{a^2-b^2}{ab} = 0 \quad \Big| : (b-a) \neq 0$$

$$\frac{z}{ab} + z^* - \frac{a+b}{ab} = 0 \quad \Big| \times ab$$

$$\boxed{z + abz^* = a+b}$$

Точка пересечения двух секущих



$$P \in (AB) \cap (CD)$$

$$(AB):\ z + abz^* = a+b$$

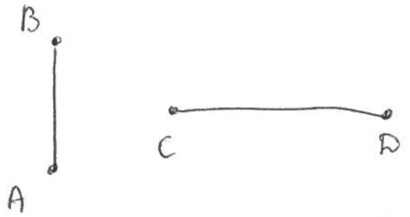
$$(CD):\ z + cdz^* = c+d$$

$$z^*(ab - cd) = (a+b) - (c+d)$$

$$ab - cd \neq 0$$

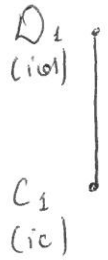
$$\boxed{p = \left(\frac{(a+b)-(c+d)}{ab-cd}\right)^*}$$

Перпендикулярность двух отрезков



$$\overline{AB} \perp \overline{CD}$$

$$\overline{AB} \parallel \overline{CD} \Leftrightarrow \frac{b-a}{d-c} \in \mathbb{R}$$

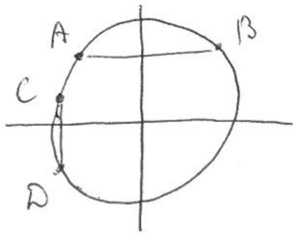


$$\frac{b-a}{i(d-c)} = it \qquad id = d_1 \quad ic = c_1$$

$$(it)^* = it$$

$$\boxed{\left(\frac{b-a}{d_1-c_1}\right)^* = -\frac{b-a}{d_1-c_1} \Leftrightarrow \left(\frac{b-a}{d_1-c_1}\right)^* + \frac{b-a}{d_1-c_1} = 0}$$

$$\left(\frac{b-a}{i(d-c)}\right)^* = -i\left(\frac{b-a}{d-c}\right)^*$$



$$A, B, C, D \in k(O; r=1)$$

$$\overline{AB} \perp \overline{CD} \Leftrightarrow \boxed{ab+cd=0}$$

$$\frac{b^*-a^*}{b-a} + \frac{d^*-c^*}{d-c} = 0 \Leftrightarrow \frac{\frac{1}{b}-\frac{1}{a}}{b-a} + \frac{\frac{1}{d}-\frac{1}{c}}{d-c} = 0$$

$$\Leftrightarrow \frac{\frac{a-b}{ab}}{b-a} + \frac{\frac{c-d}{cd}}{d-c} = 0 \Leftrightarrow \frac{1}{ab} + \frac{1}{cd} = 0 \Leftrightarrow \frac{1}{ab} = -\frac{1}{cd}$$

$$ab = -cd$$

$$\overline{AB} \perp \overline{AC}$$

$$D_1 = A$$

$$\left(\frac{b-a}{a-c}\right)^* + \frac{b-a}{a-c} = 0 \quad \Big| \cdot a-c \quad \Big| \cdot (a-c)^*$$

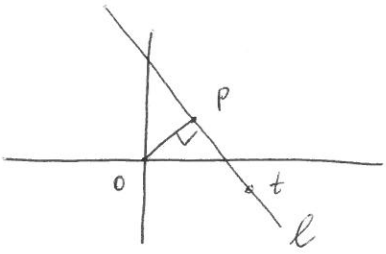
$$\boxed{(b-a)^*(a-c) + (b-a)(a-c)^* = 0}$$

↳ условие того, что в $\triangle ABC$ угол $\angle A$ — прямой

$A = O$
$a = 0$

$$\boxed{b^*c + bc^* = 0}$$

↳ условие того, что между $\overline{OB}$ и $\overline{OC}$ прямой угол



$l: \overline{ZP} \perp \overline{OP}$

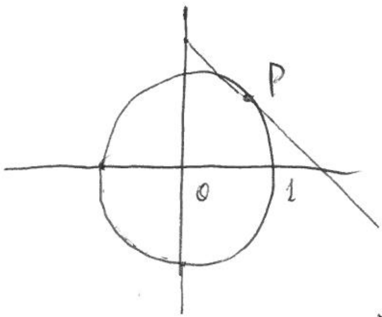
$\overline{PZ} \perp \overline{PO}$

$$\begin{pmatrix} A \leftrightarrow P \\ B \leftrightarrow Z \\ C \leftrightarrow O \end{pmatrix}$$

$$(z-p)^* p + (z-p) p^* = 0$$

$$z^* p - p^* p + z p^* - p p^* = 0$$

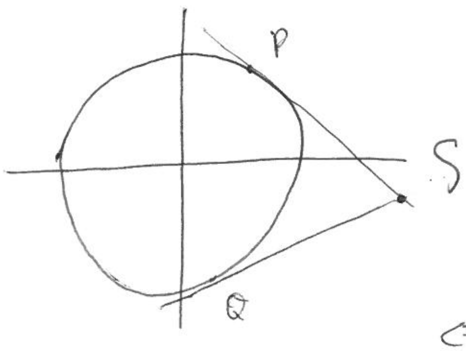
$$\boxed{z^* p + z p^* = 2 p p^* = 2|p|^2}$$



Уравнение касательной, проходящей через заданную окружность

$$p \in K(0; r=1) \iff |p| = 1: \boxed{z p^* + z^* p = 2}$$

Точка пересечения двух касательных



$$\begin{aligned} z p^* + z^* p &= 2 \quad \Big| \times q \\ z q^* + z^* q &= 2 \quad \Big| \times p \end{aligned} \Rightarrow$$

$$\Rightarrow z(p^* q - p q^*) = 2(q - p)$$

$$p^* q - q^* p \neq 0 \iff p^* q \neq q^* p \iff$$

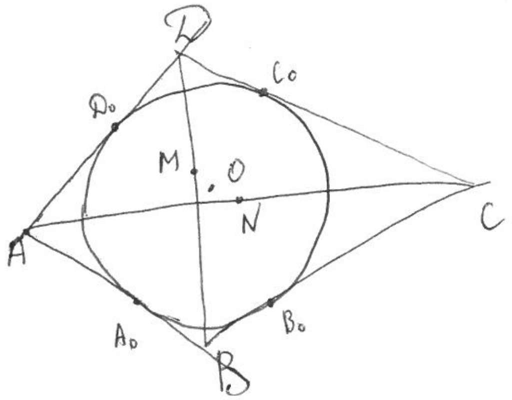
$$\iff \frac{p}{q} \neq \left(\frac{p}{q}\right)^*$$

$$\frac{p}{q} \notin \mathbb{R} \iff \frac{p-0}{q-0} \notin \mathbb{R} \iff \overline{OP} \nparallel \overline{OQ}$$

1

Дата: 26.09.16 Летописец: Людмила Баранова

Теорема Ньютона



$$\left.\begin{aligned} BM = MD \\ AN = NC \end{aligned}\right\} \Rightarrow M, O, N \in \ell$$

1. $K(O,1)$ $A_0(a)$ $B_0(b)$ $C_0(c)$ $D_0(d)$ 2. AB, BC, CD, DA – дотичні 3. $A(\ ), B(\ ), C(\ ), D(\ )$ $AB \cap DA = A$ 4. $A(\ ), C(\ ) \to N(n)$ $B(\ ), D(\ ) \to M(m)$ 5. $M, O, N \in \ell \Leftrightarrow$	П Л А Н

$$AB: \quad za^* + z^*a = 2$$
$$BC: \quad zb^* + z^*b = 2$$
$$CD: \quad zc^* + z^*c = 2$$
$$DA: \quad zd^* + z^*d = 2$$

$$\begin{cases} za^* + z^*a = 2 \ /\cdot d \\ zd^* + z^*d = 2 \ /\cdot a \end{cases} \qquad \begin{cases} za^*d + z^*ad = 2d \\ zad^* + z^*ad = 2a \end{cases}$$

$$z(a^*d - d^*a) = 2d - 2a \qquad z = \frac{2d-2a}{a^*d - d^*a} \qquad a^* = \frac{1}{a}, \quad d^* = \frac{1}{d}$$

$$\left(a^*d = d^*a \Rightarrow \frac{a}{d} = \frac{a^*}{d^*} \Rightarrow A, D, O \in \ell\right)$$

$$A\left(\frac{2ad}{a+d}\right) \quad B\left(\frac{2ab}{a+b}\right) \quad C\left(\frac{2bc}{b+c}\right) \quad D\left(\frac{2cd}{c+d}\right)$$

$$n = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}ad}{a+d} + \frac{2bc}{b+c}\right) = \frac{abd + acd + abc + bcd}{(a+d)(b+c)}$$

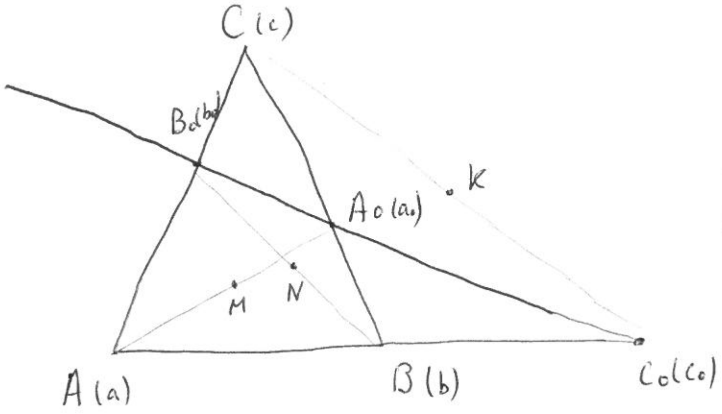
$$m = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}ab}{a+b} + \frac{\cancel{2}cd}{c+d}\right) = \frac{abc + abd + acd + bcd}{(a+b)(c+d)}$$

$$\frac{m}{n} = \frac{\cancel{(abc + abd + acd + bcd)}(a+d)(b+c)}{(a+b)(c+d)\cancel{(abd + abc + acd + bcd)}} = \frac{(a+d)(b+c)}{(a+b)(c+d)}$$

$$\frac{m^*}{n^*} = \frac{((a+d)(b+c))^*}{((a+b)(c+d))^*} = \frac{\left(\frac{1}{a} + \frac{1}{d}\right)\left(\frac{1}{b} + \frac{1}{c}\right)}{\left(\frac{1}{a} + \frac{1}{b}\right)\left(\frac{1}{c} + \frac{1}{d}\right)} = \frac{\frac{a+d}{\cancel{ad}} \cdot \frac{b+c}{\cancel{bc}}}{\frac{a+b}{\cancel{ab}} \cdot \frac{c+d}{\cancel{cd}}} = \frac{m}{n}$$

$\boxed{//}$

Теорема Гаусса



$AM = MA_0$ $M \in AA_0$
$BN = NB_0$ $N \in BB_0$
$CK = KC_0$ $K \in CC_0$

$\boxed{M, N, K \in h}$

План

2\. 1) $a(b^* - c_0^*) + b(c_0^* - a^*) + c_0(a^* - b^*) = 0$
2) 3) 4)

3\. M( ) N( ) K( )

4\. $m(n^* - k^*) + n(k^* - m^*) + k(m^* - n^*) = 0$

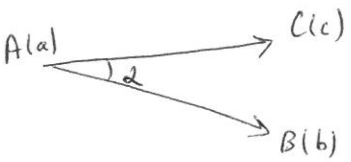
$a(b^* - c_0^*) + b(c_0^* - a^*) + c_0(a^* - b^*) = 0$
$b(a_0^* - c^*) + a_0(c^* - b^*) + c(b^* - a_0^*) = 0$
$c(b_0^* - a^*) + b_0(a^* - c^*) + a(c^* - b_0) = 0$
$c_0(a_0^* - b_0^*) + a_0(b_0 - c_0) + b_0(c_0 - a_0) = 0$

$M\left(\frac{a+a_0}{2}\right)$ $N\left(\frac{b+b_0}{2}\right)$ $K\left(\frac{c+c_0}{2}\right)$

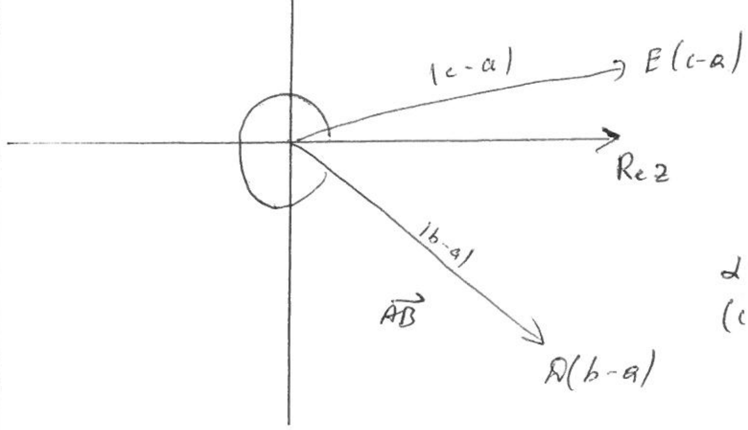
$m(n^* - k^*) + n(k^* - m^*) + k(m^* - n^*) = 0$

$(a+a_0)(b^* + b_0^* - c^* - c_0^*) + (b+b_0)(c^* + c_0^* - a^* - a_0^*) + (c+c_0)(a^* + a_0^* - b^* - b^*) = 0$

$(a+a_0)((b^* - c_0^*) - (c^* - b_0^*)) + (b+b_0)((c_0^* - a^*) - (a_0^* - c^*)) + (c+c_0)((a^* - b^*) - (b_0^* - a_0^*)) = 0$



$\varphi = \arg(b-a)$



$b - a = |b-a| \cdot e^{i\varphi}$
$c - a = |c-a| \cdot e^{i\psi}$

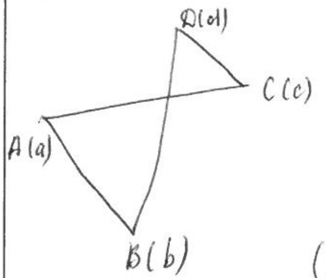
$\frac{c-a}{b-a} = \frac{|c-a|}{|b-a|} \cdot e^{i(\psi - \varphi)} = \frac{|c-a|}{|b-a|} e^{i\alpha}$

$\alpha = \arg\left(\frac{c-a}{b-a}\right) = \arg(c-a) - \arg(b-a) \pmod{2\pi}$

$(c, b, a) = \frac{c-a}{b-a}$

Дата: 26.09.16 Летописец: Людмила Баранова

$a, b, c \in \ell \Leftrightarrow (c, b; a) \in \mathbb{R}$ $(\alpha = 0$ oder $\pi)$

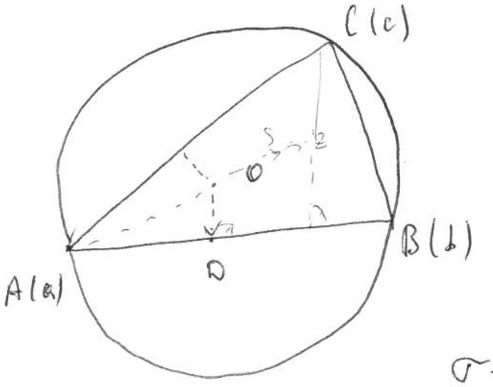


$$\alpha = \arg\left(\frac{c-a}{b-a}\right) \qquad \delta = \left(\frac{c-d}{b-d}\right)$$

$$-\alpha = \delta\ ? \Leftrightarrow A, B, C, D \in K$$

$$\alpha - \delta = 0 = \arg\left(\frac{c-a}{b-a}\right) - \arg\left(\frac{c-d}{b-d}\right) = \arg\left(\frac{c-a}{b-a} \Big/ \frac{c-d}{b-d}\right)$$

$$(c; b; a, d) = \frac{c-a}{b-a} \Big/ \frac{c-d}{b-d}$$ — двойное отношение



1) $\overline{AD} = \overline{DB}$

$D\left(\frac{a+b}{2}\right)$

2) $S\left(\frac{a+b+c}{3}\right)$

3) $\sigma = a+b+c \to \Sigma(\sigma)$

$\sigma = 3 \cdot \frac{a+b+c}{3}$

$\sigma - a = b + c$

$|\sigma - a| = |b+c| = 2 \cdot \left|\frac{b+c}{2}\right| = 2\,|\overline{OD}|$

$\overline{C\Sigma} \perp \overline{AB}$ $\qquad \overline{C\Sigma} = 2\overline{OD} \Leftrightarrow \overline{C\Sigma} \parallel \overline{OD}$

$\overline{OD} \perp \overline{AB} \Rightarrow \overline{C\Sigma} \perp \overline{AB}$ $\qquad \sigma - a = b + c$

$\sigma - b = a + c$

$\Sigma \in$ высоте из т. C

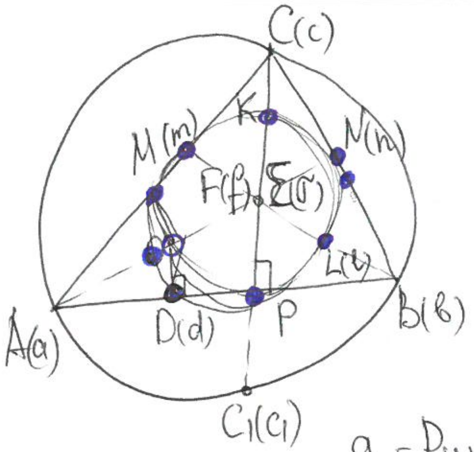
$\Sigma(\sigma = a+b+c)$ – ist Orthozentr des Δ-eckes

$O(0)$ $S\left(\frac{a+b+c}{3} = s\right)$, $\Sigma(\sigma = a+b+c) \Rightarrow O, S, \Sigma \in \ell$ – прямая Эйлера

$\overline{O\Sigma} = 3\overline{OS}$

$|\overline{OS}| : |\overline{O\Sigma}| = 1:3$

Дата: 27.09.16 Летописец: Чернякова Лиза



$K(0; r=1)$

$d = \frac{a+b}{2}$

$\sigma = a+b+c$

$f = \frac{1}{2}(a+b+c)$

$m = \frac{a+c}{2}$

$n = \frac{b+c}{2}$

$k = \frac{c+\sigma}{2}$

$l = \frac{b+\sigma}{2}$

9-Punkten-Kreis (Feuerbachsche Kreis)

1. $|\overline{DF}| = |f-d| = |\frac{1}{2}(a+b+c) - \frac{1}{2}(a+b)| = \frac{|c|}{2} = \frac{1}{2}$

$|\overline{FM}| = |\overline{FN}| = \frac{1}{2}$

2. $|\overline{KF}| = |f-k| = |\frac{1}{2}\sigma - \frac{c+\sigma}{2}| = \frac{|c|}{2} = \frac{1}{2}$

$|\overline{FL}| = |l-f| = \frac{|b|}{2} = \frac{1}{2}$

3. $|\overline{FP}|$ $|\overline{A\Sigma}| = |\overline{AC_1}|$ $|\overline{FP}| = |p-f| = |\frac{1}{2}\sigma - \frac{ab}{c})| = \frac{1}{2}$

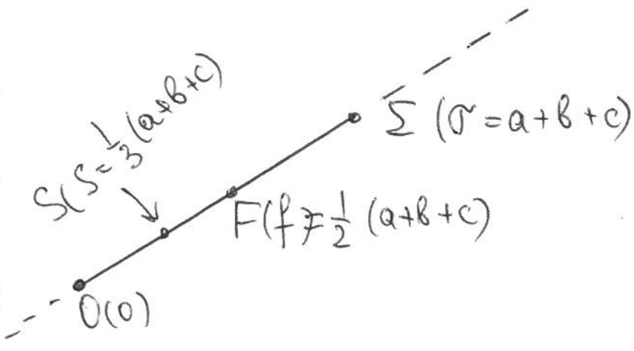
$\Downarrow$

P_p $\boxed{p = \frac{\sigma + c_1}{2} = \frac{1}{2}(\sigma - \frac{ab}{c})}$

$\left.\begin{array}{l} AB \perp CC_1 \\ A, B, C, C_1 \in K(0;1) \end{array}\right\} \Leftrightarrow ab + cc_1 = 0 \Rightarrow \boxed{c_1 = -\frac{ab}{c}}$

$|\overline{A\Sigma}| = |\sigma - a| = |b+c|$

$|\overline{AC_1}| = |c_1 - a| = |-\frac{ab}{c} - a| = \frac{|a|\,|b+c|}{|c|} = |b+c|$



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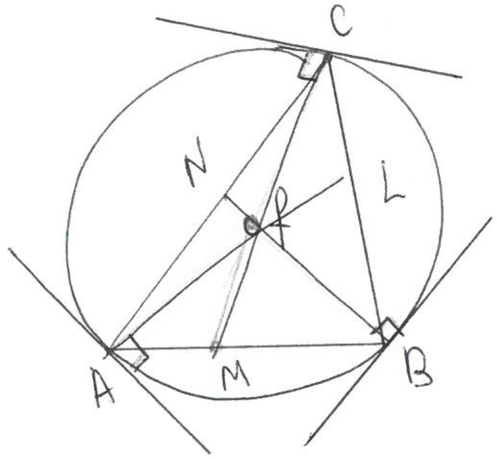
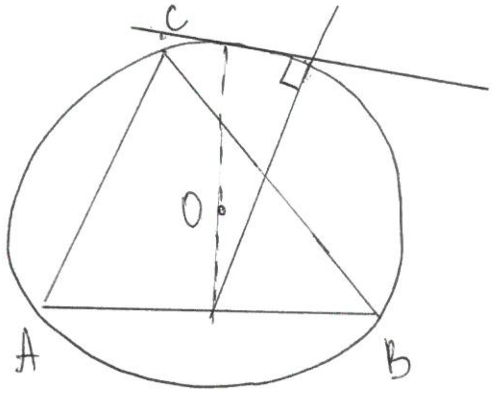
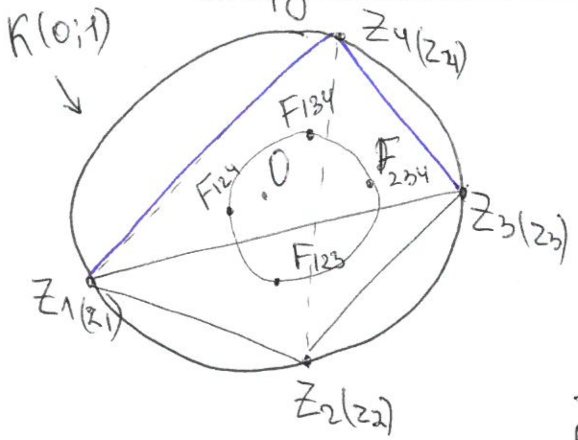




$$l_1: z = \frac{a+b}{2} + tc$$

$$t = \frac{1}{2} : f = \frac{a+b}{2} + \frac{1}{2}c$$

Окружность Ейлера



$\Delta z_1 z_2 z_3$

$\Delta z_1 z_2 z_4$

$\Delta z_1 z_3 z_4$

$\Delta z_2 z_3 z_4$

$$f_{124} = \frac{1}{2}(z_1 + z_2 + z_4)$$
$$f_{134} = \frac{1}{2}(z_1 + z_3 + z_4)$$
$$f_{123} = \frac{1}{2}(z_1 + z_2 + z_3)$$
$$f_{234} = \frac{1}{2}(z_2 + z_3 + z_4)$$

$$f_{1234} = \frac{1}{2}(z_1 + z_2 + z_3 + z_4)$$
$$|f_{1234} - f_{123}| = \frac{1}{2}|z_4| = \frac{1}{2}$$
$$|f_{1234} - f_{234}| = \left|\frac{1}{2} z_1\right| = \frac{1}{2}$$
$$F_{123}, F_{124}, F_{134}, F_{234} \in K\left(F_{1234}; r = \frac{1}{2}\right)$$
$$f_{12345} = \frac{1}{2}(z_1 + z_2 + z_3 + z_4 + z_5)$$

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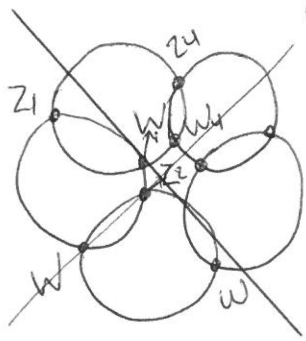
$$(a,b,c) = \frac{a-c}{b-c} \in \mathbb{R} \iff A,B,C \in l$$

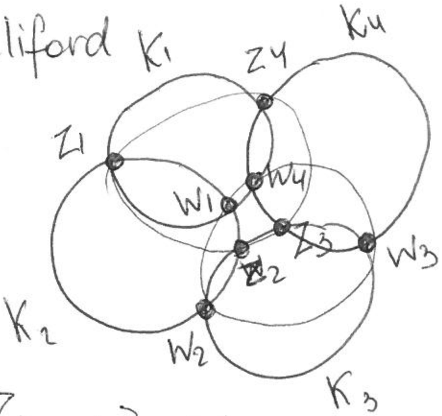
$$(a;b;c;d) = \frac{a-c}{b-c} \Big/ \frac{a-d}{b-d} \in \mathbb{R} \iff A,B,C,D \in l/K = (a,b,c)/(a,b,d)$$

$$\frac{a-d}{b-d} = \frac{\cancel{d}\left(\frac{a}{d}-1\right)}{\cancel{d}\left(\frac{b}{d}-1\right)} = \frac{\frac{a}{d}-1}{\frac{b}{d}-1} \xrightarrow[d\to\infty]{} 1$$

$$(a;b;c;d) \xrightarrow[d\to\infty]{} (a;b;c) = (a,b,c,\infty)$$

Cliford





$Z_1\ Z_2\ Z_3\ Z_4 \in K_5$
$\iff$
$W_1\ W_2\ W_3\ W_4 \in K_6$

$$(Z_1;W_2;Z_2;W_1) = \frac{Z_1-Z_2}{W_2-Z_2} \Big/ \frac{Z_1-W_1}{W_2-W_1} \in \mathbb{R}$$

$$(Z_2;W_3;Z_3;W_2) = \frac{Z_2-Z_3}{W_3-Z_3} \Big/ \frac{Z_2-W_2}{W_3-W_2} \in \mathbb{R}$$

$$(Z_3;Z_4;W_4;W_3) = \frac{Z_3-Z_4}{W_4-Z_4} \Big/ \frac{Z_3-W_3}{W_4-W_3} \in \mathbb{R}$$

$$(Z_4;W_1;Z_1;W_4) = \frac{Z_4-Z_1}{W_1-Z_1} \Big/ \frac{Z_4-W_4}{W_1-W_4} \in \mathbb{R}$$

$$\frac{(Z_1;W_2;Z_2;W_1)}{(Z_2;W_3;Z_3;W_2)} \cdot \frac{(Z_3;Z_4;W_4;W_3)}{(Z_4;W_1;Z_1;W_4)} = \frac{\frac{Z_1-Z_2}{W_2-Z_2} \Big/ \frac{Z_1-W_1}{W_2-W_1}}{\frac{Z_2-Z_3}{W_3-Z_3} \Big/ \frac{Z_2-W_2}{W_3-W_2}} \times$$

$$\times \frac{\frac{Z_3-Z_4}{W_4-Z_4} \Big/ \frac{Z_3-W_3}{W_4-W_3}}{\frac{Z_4-Z_1}{W_1-Z_1} \Big/ \frac{Z_4-W_4}{W_1-W_4}}$$

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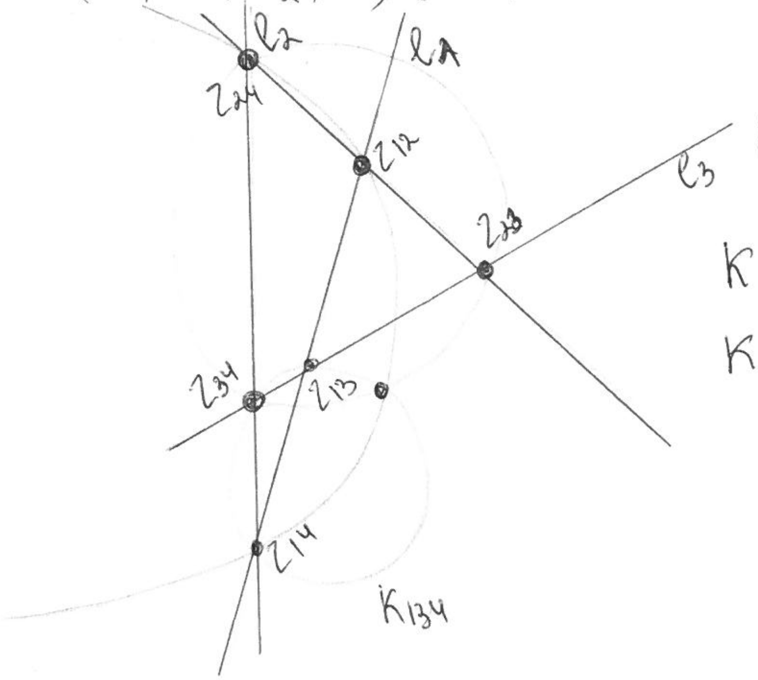
$$\frac{(z_1-z_2)(w_2-w_1)(z_3-z_4)(w_4-w_5)(w_3-z_3)(z_2-w_2)(w_1-z_1)(z_4-w_4)}{(w_2-z_2)(z_1-w_1)(w_4-z_4)(z_3-w_3)(z_2-z_3)(w_3-w_2)(z_4-z_1)(w_1-w_4)}=$$

$$=\frac{(z_1-z_2)(z_3-z_4)(w_2-w_1)(w_4-w_3)}{(z_2-z_3)(z_4-z_1)(w_3-w_2)(w_1-w_4)}=$$

$$=\pm(z_1;z_3;z_2;z_4)(w_1;w_2;w_2;w_4)\in\mathbb{R}$$



K_{234} l_2 : z_{24} z_{25}

l_2 l_1 : ∞ z_{12}

K_{134} z_{13} z_{14}

K_{134} K_{234} z_{34}

Комплексные числа в алгебре

$5+x=3$ $x=-2$ | $(x+2)^2=0$

$x^2=2$ | $(x+2)(x+2)=0$

| -2 -2

$x^2+1=0$

$x=\pm i$

$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n=0$ Основная теорема алгебры

$\{a_0;a_1;\ldots;a_n\}\subset\mathbb{C}$

a $P(x)$ $P(a)=0$

$P(x)=(x-a)Q(x)$

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$z^n + a_1 z^{n-1} + a_2 z^{n-2} + \ldots + a_{n-1} z + a_n = 0$

$(z - z_1)(z^{n-1} + b_1 z^{n-2} + \ldots + b_{n-2} z + b_{n-1}) = 0$

$z^n + a_1 z^{n-1} + \ldots + a_{n-1} z + a_n = (z - z_1)(z - z_2)\ldots(z - z_n)$

$z^n + a_1 z^{n-1} + \ldots + a_{n-1} z + a_n = 0 \quad (*)$

$\{a_1; \ldots; a_n\} \in \mathbb{R}$

$\boxed{\begin{matrix} a^* = a \\ a \in \mathbb{R} \end{matrix}}$

Теорема: z, z^*

Дов-ння: $(z_0^n + a_1 z_0^{n-1} + \ldots + a_n)^* = 0^*$

$(z_0^*)^n + a_1 (z_0^*)^{n-1} + \ldots + a_n = 0$

z_0^*

$P(z) = \underbrace{(z - z_0)(z - z_0^*)}_{z^2 - (z_0 + z_0^*) z + z_0 z_0^*} Q(z)$

$z_0 \; z_0^*$

$z_0 = a + bi$

$z_0^* = a - bi$

$z_0 + z_0^* = 2a \in \mathbb{R}$

$z_0 z_0^* = a^2 + b^2 \in \mathbb{R}$

Д/з

$\boxed{a^6 + 1}$